Tradeoff Analysis of Throughput and Fairness on CDMA Packet Downlinks With Location-Dependent QoS

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***Abstract*—This paper presents the quantitative tradeoff relation between system throughput and fairness on multirate code-division multiple-access (CDMA) downlinks. We develop and use an analytical model and a method for performance evaluation. The proposed model and method reflect the effects of multiplexing scheme, a limited data-rate set, log-normal shadowing, the best base station selection, and self-interference. System performance measures include *asymptotic throughput*, *fairness performance factors*, and *outage probability*. The performance of packet channels for delay-tolerant service is evaluated based on 3GPP wide-band code-division multiple-access (WCDMA) system model. Using the method developed, various algorithms for time resource assignment are applied and the system performance measures are derived and compared. We have derived the tradeoff between asymptotic throughput and fairness performance factors caused by the location-dependent carrier-to-interference ratio. The results show that we can control the tradeoff by applying various time resource-assignment schemes. It is also shown that the tradeoff in the system with the limited data-rate set can be improved by setting the number of simultaneous users properly.**

***Index Terms*—Code-division multiplexing (CDM), code-division multiple access (CDMA), downlink, fairness, multirate transmission, quality of service (QoS), throughput, time-division multiplexing (TDM), tradeoff.**

I. Introduction

WITH A major shift from speech services to Internet services expected in wireless communication industries, there is an urgent need for a control scheme that can efficiently handle packet-data traffic. Compared with speech service, Internet services have different requirements for quality-of-service (QoS) and traffic characteristics. Most require a lower bit-error rate (BER), but tolerate a longer delay. Different QoS levels within a certain range are also allowable for the same kind of service, i.e., best-effort-type services. Moreover, their traffic is bursty and they set more capacity requirements for downlink than uplink. The control schemes of packet transmission should be designed to meet the requirements discussed previously.

One of the unique concerns in the radio-access network (RAN) is the tradeoff between throughput and fairness on the downlink. System throughput can be maximized by allocating more radio resources to a user with higher carrier-to-interference ratio (C/I), e.g., a user close to the base station (BS). However, a user with lower C/I will have proportionately higher latency. Even for the same fraction of available power on the multirate code-division multiple-access (CDMA) downlink, higher data rate may be assigned to the user with higher C/I [1], [2]. On the other hand, if more power or service periods are assigned to the lower C/I user for fairness, system throughput will be decreased. Therefore, it is not easy to achieve the objective that a fair QoS is provided over the whole service area and the system throughput is maximized at the same time. This tradeoff caused by location-dependent QoS is an inherent feature of RAN. In order to develop an efficient packet-transmission scheme for downlink, we first need to investigate the tradeoff between throughput and fairness.

Throughput and fairness can be affected by the multiplexing scheme applied. In multirate CDMA systems, radio resources for the packet channel can be assigned in a code- or time-division manner [3]. As a BS transmits to a greater number of users simultaneously by code-division multiplexing (CDM), the assigned data rate to a user is decreased, but the service time per user may be increased. The allowable data rates in practical systems are set as a number of discontinuous values and also limited to a peak rate.[1] As the allowed range for the data rate becomes narrower, fairness performance might be improved. However, if the number of simultaneous users is set to be too small, the system throughput can be determined by the peak data-rate limitation, rather than the interference power. On the contrary, if the number of simultaneous users is set to be too large, the assigned power per user will be so low that even the minimum data rate is not assigned to the user at a poor C/I region, i.e., an outage event occurs. Therefore, we should cautiously apply the multiplexing scheme with considering the limited data-rate set.

There have been a number of papers that mention fairness in CDMA RAN. However, a few showed the quantitative tradeoff relation between throughput and fairness, and most of the presented results were based on *Monte Carlo* computer simulations [1], [4]. Besides, the effect of the limited data-rate set and

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[1]In this paper, we refer to a finite set of data rates supported by physical channel as a *limited data-rate set*.

the multiplexing scheme on the tradeoff has not been reported. Holtzman [5] investigated a proportional fair algorithm that can strike a good compromise between throughput and fairness with the help of a fast scheduling. Although a throughput difference between two specific users with different C/Is was shown in [5], the tradeoff relation for randomly distributed users was not presented. Bharghavan *et al.* [6] introduced and compared several contemporary research efforts on adapting fair queueing algorithms of wired networks to the wireless domain. However, the tradeoff on the CDMA downlinks has not been treated in most studies of wireless fair queueing. Meanwhile, analytical approaches for throughput evaluation of the multirate CDMA downlinks did not carefully consider RAN features such as the best BS selection on shadowed radio paths or self-interference in multipath fading [7], [8].

This paper presents the quantitative tradeoff relation between throughput and fairness caused by the location-dependent C/I on multirate CDMA downlinks. We focus on the downlink features of CDMA RAN, rather than the characteristics of the bursty traffic. Hence, we assume a system with fully loaded cells where a number of users have data to send all the time. Our contributions are threefold: 1) analytical methodology for the tradeoff analysis of throughput and fairness; 2) a downlink system model including various features, such as the limited data-rate set, log-normal shadowing, the best BS selection, and self-interference; and 3) quantitative effects of the limited data-rate set, multiplexing scheme, and time-resource-assignment algorithm on the tradeoff.

In the next section, we set the system model in consideration of various features of the CDMA downlink. In Section III, we develop an analytical method for the performance evaluation in terms of throughput and fairness. The numerical results in a 3GPP WCDMA system model are shown in Section IV. Finally, we draw the conclusion in Section V.

II. System Model

We describe power allocation and data-rate assignment in the system model and propose the received bit energy per noise and interference power density (E_b/N_t) model, including the impact of self interference. Considering the best BS selection, we develop the statistical model of the other cell interference and formulate the distribution of users that are connected to a BS.

A. Downlink Packet Transmission

The downlink in the system model is comprised of three channel types: overhead, circuit, and packet. The overhead channel is used to broadcast control information within a cell, e.g., common pilot channel (CPICH) and broadcast channel (BCH) in 3GPP system [3]. All BSs have the same maximum transmission power P^M. A fixed portion α of P^M, i.e., $P^O = \alpha P^M$, is allocated to the overhead channels. For simplicity, total power of all the circuit channels for delay-sensitive services is assumed to be a fixed portion δ of the current total BS transmission power P_i^C, i.e., $P_i^{\mathrm{Ckt}} = \delta P_i^C$. Then, P_i^C can be represented by

$$P_i^C = \sum_{j=0}^{N-1} \beta_{i,j} P_{i,j}^U + P^O + P_i^{\mathrm{Ckt}} \tag{1}$$

where subscripts i and j denote the ith BS (BS_i) and the jth mobile station (MS_j), respectively. $P_{i,j}^U$ is the maximum allowable power for the MS_j, $\beta_{i,j}$ denotes the actual portion of $P_{i,j}^U$ for transmission $(0 < \beta_{i,j} \leq 1)$, and N is the number of data users receiving simultaneously.

The packet channel carries dedicated user data for delay-tolerant services. We apply the simple power-allocation scheme such that the remaining BS power after allocating P^O and P_i^{Ckt} is equally divided among packet data users receiving simultaneously. Therefore, $P_{i,j}^U$ for every user is the same as

$$P_{i,j}^U = P_i^U = \frac{P^M - P^O - P_i^{\mathrm{Ckt}}}{N}. \tag{2}$$

The CDM and TDM schemes are applied for the packet transmission. As in [3], the packet transmissions with the CDM and with the TDM are hereafter called the code-division scheduling (CDS) and the time-division scheduling (TDS), respectively. The combined scheme can be applied by adjusting N, which also denotes the number of the code-division multiplexed users. The multiplexing scheme with $N = 1$ can be regarded as pure TDS. As N is increased, the scheme works more like CDS.

The data rate assigned to a user is dependent on the received C/I. The highest available data rate is assigned to each user as long as the received E_b/N_t remains above the required value at its location with the given power P_i^U. A rate-control scheme can be applied to moving users, which permits a data-rate change in every frame [9]. The fast closed-loop power control is also applied to the packet channels [3], [10]. While the rate-control scheme may assign the different data-rate frame by frame, according to the user's location, power control adjusts the transmission power slot by slot for achieving the E_b/N_t target.[2] We assumed that the received E_b/N_t is to be kept at its target by the perfect operation of the power and rate controls. If any rate cannot be assigned because of a poor radio condition, an outage event occurs. When the outage occurs, the transmission opportunity and power can be reallocated to another user that has a good radio condition.

We illustrate the power and rate assignment with an example in Fig. 1. In the nth frame, the data-rate $R1$ is assigned to a user, because P_i^U is lower than the required power for $R2$ but higher than that for $R1$. Note that P_i^U is the maximum allowable level for the averaged transmission power over one frame. In this figure, the dotted line denotes the instantaneous power level that is adjusted by the fast power control and N_s denotes the number of slots per frame. The averaged transmission power over the nth frame becomes $\beta_{i,j} P_i^U$ due to the power-control

[2]Since this paper focuses on the tradeoff between throughput and fairness caused by the location-dependent C/I, we do not consider a fast scheduling that can swap the data transmission between temporary faded users and can also adjust the data rate slot by slot. However, we expect that the analytical model and method to be shown in this paper could be extended to an analysis with the fast scheduling coping with fast fading.

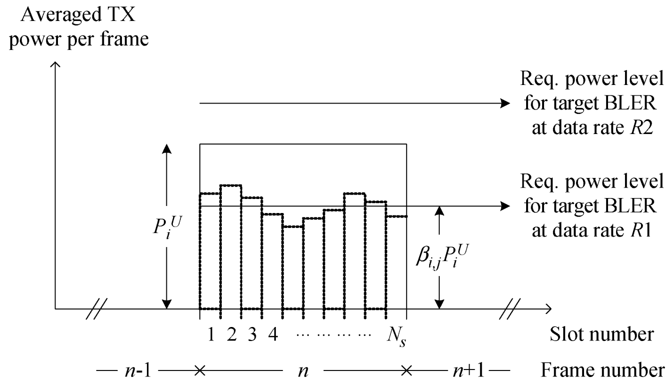


Fig. 1. Example of power and rate assignment.

operation. Hence, the averaged transmission power per frame is always equal to or smaller than P_i^U.

An ideal automatic retransmission query (ARQ) procedure is assumed as follows. The ARQ mechanism ensures retransmission of transport blocks in error, the number of repeat is unlimited, and error rate of transport block, i.e., block error rate (BLER), is kept at the required value by the power and rate controls.

B. Received E_b/N_t Model

Multirate services with high data rate can be implemented by variable spreading factor (VSF) or multicode (MC) schemes [11]. Despite employing orthogonal spreading codes on the downlink, the lack of orthogonality in the multipath fading channels can produce multiuser interference in the same cell. Self-interference within the packet channel of a user is also generated due to small spreading factor (SF) or many MC channels used for high-rate transmission [12]. Even though the self-interference is considered in many link-level studies, as in [11] and [12], it has been neglected in many system-level studies [7], [8]. For the exact evaluation of throughput on downlink, the effects of self-interference as well as multiuser interference must be taken into consideration.

We present the received E_b/N_t model, including the impact of the self-interference. The proposed model can be applied to both MC and VSF systems. For the MC system, considering the self-interference between MC channels, the received E_b/N_t of a code channel can be modeled as

$$\left(\frac{E_b}{N_t}\right)_{i,j} = \frac{\beta_{i,j}P_{i,j}^{\mathrm{Code}}}{I_{i,j}^{SF}+I_{i,j}^{SC}+I_{i,j}^{OC}}\cdot\frac{W}{R_0} \tag{3}$$

where spreading bandwidth W is assumed to be the chip rate and R_0 denotes the data rate of a code channel. The maximum allowable power per code channel $P_{i,j}^{\mathrm{Code}}$ is given by

$$P_{i,j}^{\mathrm{Code}} = \frac{P_i^U}{k_{i,j}} \tag{4}$$

where $k_{i,j}$ is the number of assigned MC channels. The maximum $k_{i,j}$ satisfying the condition below is assigned to the user.

$$\frac{\frac{\beta_{i,j}P_i^U}{k_{i,j}}}{I_{i,j}^{SF}+I_{i,j}^{SC}+I_{i,j}^{OC}}\cdot\frac{W}{R_0} \geq \left(\frac{E_b}{N_t}\right)_{\mathrm{REQ}} \tag{5}$$

where $(E_b/N_t)_{\mathrm{REQ}}$ is the required value for a target BLER at the data rate R_0. Then, the aggregate data rate for user $R_{i,j}$ is given by

$$R_{i,j} = k_{i,j}\cdot R_0. \tag{6}$$

Total interference can be divided into the self-interference $I_{i,j}^{\mathrm{SF}}$, same-cell interference $I_{i,j}^{\mathrm{SC}}$, and other-cell interference $I_{i,j}^{\mathrm{OC}}$. In (3), thermal noise is assumed to be negligible. $I_{i,j}^{\mathrm{SF}}$ denotes the interference between the MC channels assigned to a user. Hence, it is proportional to $k_{i,j}$. $I_{i,j}^{\mathrm{SC}}$ is the sum of the interferences from the other packet, overhead, and circuit channels within the same cell. $I_{i,j}^{\mathrm{OC}}$ is total interference from other BSs. These can be expressed as

$$I_{i,j}^{\mathrm{SF}} = (1-\phi^{\mathrm{SF}})\beta_{i,j}(k_{i,j}-1)P_{i,j}^{\mathrm{Code}} \tag{7}$$

$$I_{i,j}^{\mathrm{SC}} = (1-\phi^{\mathrm{SC}})\left(\sum_{n=0,n\neq j}^{N-1}\beta_{i,n}P_i^U + P^O + P_i^{\mathrm{Ckt}}\right) \tag{8}$$

$$I_{i,j}^{\mathrm{OC}} = \sum_{m=0,m\neq i}^{M}\sum_{n=0}^{N-1}\left(\beta_{m,n}P_m^U + P^O + P_m^{\mathrm{Ckt}}\right)\frac{L_{m,j}}{L_{i,j}} \tag{9}$$

where M is the number of BSs near BS_i.

In (7) and (8), ϕ^{SF} and ϕ^{SC} denote the orthogonality factor between MC channels. An orthogonality factor of 1 corresponds to perfectly orthogonal code channels, while with the factor of 0, the orthogonality is lost. A link-level study [12] showed that the lack of orthogonality between code channels of a user is similar to that between other user code channels. Therefore, the two values can be approximated to be identical as

$$\phi \approx \phi^{\mathrm{SF}} \approx \phi^{\mathrm{SC}}. \tag{10}$$

In (9), $L_{i,j}$ denotes the path gain. At distance $r_{i,j}$ from BS_i, the propagation loss is inversely proportional to

$$L_{i,j} \equiv r_{i,j}^{-\mu}10^{\frac{(a\hat{\xi}_j+b\xi_i)}{10}} \tag{11}$$

where μ is the path-loss exponent. $\hat{\xi}_j$ and ξ_i denote the MS_j shadowing and the BS_i shadowing, respectively. Both are *Gaussian* random variables with zero mean and standard deviation σ. In this model, site-to-site correlation is applied as $a^2 = 1 - b^2$ [13].

The E_b/N_t model shown previously can also be applied to the VSF system, because it was reported that the link-level performance with the VSF scheme is approximately equal to that with the MC scheme [12]. For the VSF systems, the minimum data rate R_0 should be set to the data rate for the largest SF. Then, R_0 and $k_{i,j}$ can be regarded as the data rate of an equivalent code channel and the number of the assigned equivalent code channels, respectively. Note that $k_{i,j}$ for VSF systems is a power of 2 because the SF in most systems is given by a power of 2.

C. Statistical Model of the Relative Other-Cell Interference

We show a statistical model of the downlink interference for the analytical performance evaluation. The received E_b/N_t on

CDMA downlink is mainly influenced by the ratio of interference from adjacent BS to that from the connected BSs. A MS in a shadowed environment is connected to the BS with the smallest propagation loss, i.e., the best BS rather than the closest BS. The best BS selection can considerably affect the statistics of the relative interference. Pratesi *et al.* [14] analyzed the cochannel interference statistics with various methods, but without considering the best BS selection. In this work, based on *Wilkinson's* approximation that is one of the well-known log-normal approximations [14], [15], the statistical model of the relative other-cell interference is developed considering the best BS selection.

We define random variables X and Z as

$$X_{i,j} \equiv \sum_{m=0,m\neq i}^{M} \frac{L_{m,j}}{L_{i,j}} = \sum_{m=0,m\neq i}^{M} Y_{i,m,j} \tag{12}$$

$$Z_{i,j} \equiv 10\log X_{i,j}. \tag{13}$$

where $Y_{i,m,j} \equiv L_{m,j}/L_{i,j} = V_{i,m,j}10^{b(\xi_m-\xi_i)/10}$ and $V_{i,m,j} \equiv (r_{i,j}/r_{m,j})^{\mu}$. Note that the distance from BS_m to MS_j, i.e., $r_{m,j}$, can be represented by a function of $(r_{i,j},\theta_{i,j})$, which are the polar coordinates with respect to BS_i. The subscripts of $(r_{i,j},\theta_{i,j})$ are hereafter dropped for convenience: i.e., $r \equiv r_{i,j}$ and $\theta \equiv \theta_{i,j}$. Note that $V_{i,m,j}$ can also be represented by a function of (r,θ).

The first and second moments of X at the given position (r,θ) can then be obtained as

$$m_{X_{i,j}|r,\theta} \equiv E[X_{i,j}|r,\theta] = \sum_{m=0,m\neq i}^{M} E[Y_{i,m,j}|r,\theta] \tag{14}$$

$$\begin{aligned}\nu_{X_{i,j}|r,\theta} &\equiv E\left[X_{i,j}^2|r,\theta\right] \\ &= \sum_{m=0,m\neq i}^{M} E\left[Y_{i,m,j}^2|r,\theta\right] \\ &\quad + \sum_{l=0,l\neq i}^{M}\sum_{m=0,m\neq i,m\neq l}^{M} E[Y_{i,l,j}Y_{i,m,j}|r,\theta].\end{aligned} \tag{15}$$

With the best BS selection, every MS is connected to the BS that has the smallest propagation loss. Therefore, the inequality $L_{i,j} > L_{m,j}$ should be met for all m, except i in (12). With this condition, the expected values in (14) and (15) can be obtained as shown in (16)–(18) at the bottom of the page, where $A_{i,n,j}(x,y)$ is defined as

$$A_{i,n,j}(x,y) \equiv \frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{bx - 10\log V_{i,n,j}}{\sqrt{2}b\sigma} - y\frac{\sigma b \ln 10}{10\sqrt{2}}\right). \tag{19}$$

In the equation above, $\mathrm{erf}(\cdot)$ denotes the error function. Note that $A_{i,n,j}(x,y)$ depends on (r,θ) because $V_{i,m,j}$ is a function of (r,θ). Although $A_{i,n,j}(x,y,r,\theta)$ is the exact notation, $A_{i,n,j}(x,y)$ is used for convenience.

As the conventional *Wilkinson's* approximation, we approximate X as a log-normal random variable. Z then becomes a Gaussian random variable with mean $m_{Z_{i,j}|r,\theta} = 10\log(m_{X_{i,j}|r,\theta}^2/\sqrt{\nu_{X_{i,j}|r,\theta}})$ and variance $\sigma_{Z_{i,j}|r,\theta}^2 = (100/\ln 10)\log(\nu_{X_{i,j}|r,\theta}/m_{X_{i,j}|r,\theta}^2)$ [15]. Zorzi [16] provided the joint probability density function (pdf) of $L_{i,j}$ for $i = 0,1,\cdots,M$, considering the best BS selection. From the provided formula, however, it is not easy to obtain a pdf of X or Z in the closed from. In our approach, although the log-normal approximation is employed and numerical integrations are involved, the pdf of Z has been obtained in the closed form of normal distribution that would be very useful for system-level analyses.[3]

D. Distribution of the Connected Users

We formulate the distribution of MSs that have been successfully connected to the BS of interest. As explained, a MS is connected to the BS with the smallest propagation loss, rather than the closest BS. For example, even if an MS is located closer to BS_{i+1} than BS_i, the MS may be connected to BS_i due to the independent shadowing of each BS and the best BS selection. Therefore, distribution of MSs connected to the BS should include the effect of the best BS selection.

[3]In regard to validation of the applied approximation, see [17].

$$E[Y_{i,m,j}|r,\theta] = V_{i,m,j}e^{\frac{\sigma^2b^2(\ln 10)^2}{200}} \cdot \frac{\int_{-\infty}^{\infty} 10^{-\frac{bx}{10}} e^{-\frac{x^2}{2\sigma^2}} A_{i,m,j}(x,1) \prod_{n=0,n\neq i,n\neq m}^{M} A_{i,n,j}(x,0)dx}{\int_{-\infty}^{\infty} e^{-\frac{x^2}{2\sigma^2}} \prod_{n=0,n\neq i}^{M} A_{i,n,j}(x,0)dx} \tag{16}$$

$$E\left[Y_{i,m,j}^2|r,\theta\right] = V_{i,m,j}^2e^{\frac{\sigma^2b^2(\ln 10)^2}{50}} \cdot \frac{\int_{-\infty}^{\infty} 10^{-\frac{bx}{5}} e^{-\frac{x^2}{2\sigma^2}} A_{i,m,j}(x,2) \prod_{n=0,n\neq i,n\neq m}^{M} A_{i,n,j}(x,0)dx}{\int_{-\infty}^{\infty} e^{-\frac{x^2}{2\sigma^2}} \prod_{n=0,n\neq i}^{M} A_{i,n,j}(x,0)dx} \tag{17}$$

$$\begin{aligned}E[Y_{i,l,j}Y_{i,m,j}|r,\theta] &= V_{i,l,j}V_{i,m,j}e^{\frac{\sigma^2b^2(\ln 10)^2}{100}} \\ &\cdot \frac{\int_{-\infty}^{\infty} 10^{-\frac{bx}{5}} e^{-\frac{x^2}{2\sigma^2}} A_{i,l,j}(x,1)A_{i,m,j}(x,1) \prod_{n=0,n\neq i,n\neq m,n\neq l}^{M} A_{i,n,j}(x,0)dx}{\int_{-\infty}^{\infty} e^{\frac{-x^2}{2\sigma^2}} \prod_{n=0,n\neq i}^{M} A_{i,n,j}(x,0)dx}\end{aligned} \tag{18}$$

Let $f(r,\theta|\mathrm{BS}_i,\mathrm{Avail})$ be the conditional joint pdf of (r,θ) on the condition that the MS located at (r,θ) is connected to the BS_i and its radio link is available. This pdf can be derived as shown in (20) at the bottom of the page, where $\Pr(\mathrm{BS}_i,\mathrm{Avail}|r,\theta)$ is the probability that MS having been located at (r,θ) is connected to the BS_i and the radio link is available. $\Pr(\mathrm{Out}|\mathrm{BS}_i,(r,\theta))$ is outage probability for the MS that has selected BS_i at (r,θ). A_0 denotes the entire region that is feasible for the selection of BS_i. $f(r,\theta)$ is the joint pdf of the MS position on the region A_0, regardless of the connected BS.

The probability of selecting BS_i at the given position (r,θ) is computed as

$$\begin{aligned}&\Pr(\mathrm{BS}_i|r,\theta)\\&\quad=\Pr(L_{i,j}>L_{0,j},L_{i,j}>L_{1,j},\ldots,L_{i,j}>L_{M,j}|r,\theta)\\&\quad=\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty}e^{-\frac{x^2}{2\sigma^2}}\prod_{n=0,n\neq i}^{M}A_{i,n,j}(x,0)dx.\end{aligned}\qquad(21)$$

Note that $A_{i,n,j}(x,0)$ depends on (r,θ), as explained in the previous section. We will show the effect of the best BS selection on the distribution of the connected users, in Section IV.

III. Throughput and Fairness Analysis

We develop an analytical method for the analysis of throughput and fairness evaluation. The distribution of the assigned data rate to a user is derived in a shadowing environment. We then define and derive various system-performance measures. The numerical formulas to be shown in this section are for VSF systems. However, they can also be applied for the analysis of the MC systems without major modifications.

A. Distribution of the Assigned Data Rate

We define $p_R(R|\mathrm{BS}_i,\mathrm{Avail})$ as the distribution of the data rate assigned to a user who has selected BS_i and had an available radio link. Subscripts i and j are dropped for convenience. Since $R=k\cdot R_0$, $p_R(R|\mathrm{BS}_i,\mathrm{Avail})$ is derived in the form of probability mass function (pmf) and is also identical with the conditional pmf of k, $p_k(k|\mathrm{BS}_i,\mathrm{Avail})$. We can obtain $p_k(k|\mathrm{BS}_i,\mathrm{Avail})$ by averaging the pmf of each position over the region A_0. Therefore,

$$p_k(k|\mathrm{BS}_i,\mathrm{Avail})=\iint_{A_0}p_k(k|\Xi)\cdot f(r,\theta|\mathrm{BS}_i,\mathrm{Avail})drd\theta\qquad(22)$$

where the event $\Xi\equiv((r,\theta),\mathrm{BS}_i,\mathrm{Avail})$ means that the user located at (r,θ) selects BS_i and has an available radio link. The pdf $f(r,\theta|\mathrm{BS}_i,\mathrm{Avail})$ was formulated in the previous section. Hence, we only have to derive the conditional pmf $p_k(k|\Xi)$. The goal of this subsection is to obtain $p_k(k|\Xi)$.

We begin by simplifying the received E_b/N_t expression, so that k can be represented as a simple function of Gaussian random variable Z. Then, we obtain $p_k(k|\Xi)$ from the pdf of Z. The outage probability is also derived from the distribution of Z.

To simplify the expression of the maximum allowable power for data user, i.e., P_i^U in (2), we assume that the random variable $\beta_{i,j}$ in (1) is a constant value $\overline{\beta}$, which is to be defined and derived in the next section. Then, P_i^{Ckt} and P_i^U can be represented by P^{Ckt} and P^U, respectively.

$$P_i^{\mathrm{Ckt}}\approx P^{\mathrm{Ckt}}\equiv\zeta P^M\qquad(23)$$

$$P_i^U\approx P^U\equiv\frac{1-\alpha-\zeta}{N}P^M\qquad(24)$$

where

$$\zeta\equiv\frac{\delta\left(\alpha(1-\overline{\beta})+\overline{\beta}\right)}{1-\delta(1-\overline{\beta})}.\qquad(25)$$

We now consider the received E_b/N_t expression in (5). Every BS tries to make the utmost use of the given power P^U in order to assign a higher data rate. Therefore, β for the user of interest in (5) and (7), i.e., $\beta_{i,j}$, should be set to 1. On the other hand, βs for the other users in (8) and (9), i.e., $\beta_{i,n}$ and $\beta_{m,n}$, are assumed to be $\overline{\beta}$ for simplicity.[4]

We are to represent the discrete random variable k as a function of the Gaussian random variable Z. Since Z is a continuous value, we need to introduce the continuous random variable of k. Let the continuous random variable of k be $\widetilde{k}$. We may replace k with $\widetilde{k}$ in (5) and invert (5) as an equality. Next, substitute P^{Ckt} for P_i^{Ckt} and P_m^{Ckt}, and P^U for P_i^U and P_m^U. Then, solving (5) for $\widetilde{k}$, we have

$$\widetilde{k}=g(Z)=\frac{\Lambda}{\Gamma+10^{Z/10}}\qquad(26)$$

where

$$\Lambda\equiv\frac{1-\alpha-\zeta}{N\left[(1-\alpha-\zeta)\overline{\beta}+\alpha+\zeta\right]}\times\left[(1-\phi)+\frac{\frac{W}{R_0}}{\left(\frac{E_b}{N_t}\right)_{\mathrm{REQ}}}\right]\qquad(27)$$

[4]The setting of $\beta_{i,j}=1$ is valid only when we find the maximum $k_{i,j}$ satisfying (5). It is because the given power to a user P^U should be utilized to the fullest for the highest possible $k_{i,j}$. However, since $k_{i,j}$ is a discrete value with a peak limitation, the actual transmission power after determining $k_{i,j}$ becomes lower than P^U, as shown in Fig. 1. When computing interference from other user signal, we should consider the actual transmission power. For this reason, we assume that βs for the other users is a constant value smaller than 1.

$$\begin{aligned}f(r,\theta|\mathrm{BS}_i,\mathrm{Avail})&=\frac{\Pr(\mathrm{BS}_i,\mathrm{Avail}|r,\theta)\cdot f(r,\theta)}{\iint_{A_0}\Pr(\mathrm{BS}_i,\mathrm{Avail}|r,\theta)\cdot f(r,\theta)drd\theta}\\&=\frac{[1-\Pr(\mathrm{Out}|\mathrm{BS}_i,(r,\theta))]\cdot\Pr(\mathrm{BS}_i|r,\theta)\cdot f(r,\theta)}{\iint_{A_0}[1-\Pr(\mathrm{Out}|\mathrm{BS}_i,(r,\theta))]\cdot\Pr(\mathrm{BS}_i|r,\theta)\cdot f(r,\theta)drd\theta}\end{aligned}\qquad(20)$$

$$\Gamma \equiv \frac{1-\phi}{(1-\alpha-\zeta)\overline{\beta}+\alpha+\zeta} \times \left[(1-\alpha-\zeta)\left(\overline{\beta}+\frac{1-\overline{\beta}}{N}\right)+\alpha+\zeta\right]. \quad (28)$$

Note that $k = 2^{\lfloor \log_2 \widetilde{k} \rfloor} = 2^{\lfloor \log_2 g(Z) \rfloor}$ for the VSF systems, where $\lfloor x \rfloor$ denotes the largest integer that is smaller than or equal to x. Considering the peak data-rate limitation, the maximum can be set to

$$k_{\max} \equiv \min\left(\frac{R_{\max}}{R_0}, 2^{\left\lfloor \lim_{Z\to-\infty} \widetilde{k} \right\rfloor}\right) = \min\left(\frac{R_{\max}}{R_0}, 2^{\lfloor \log_2 \frac{\Lambda}{\Gamma} \rfloor}\right) \quad (29)$$

where $R_{\max}$ is the maximum allowable data rate per packet channel.

As explained earlier, we apply an outage-handling scheme that reallocates the transmission opportunity of outage user to another user that has a good radio condition. Since $p_k(k|\Xi)$ is for a user that has an available radio link, we must consider Z for the nonoutage user. The outage events occurs when $\widetilde{k}$ is smaller than its minimum value $k_{\min}$, where $k_{\min}$ is determined by the given rate set. From (26), this outage condition can be rewritten as

$$Z > Z_{\max} \equiv 10\log\left(\frac{\Lambda}{k_{\min}} - \Gamma\right). \quad (30)$$

We can, therefore, define Z for nonoutage user as $\widehat{Z}$, such that $-\infty < \widehat{Z} \le Z_{\max}$. Its pdf then is

$$f_{\widehat{Z}}(z) = f_Z(z|z \le Z_{\max}) = \begin{cases} \frac{\varepsilon}{\sqrt{2\pi}\sigma_{Z|r,\theta}} \exp\left(-\frac{(z-m_{Z|r,\theta})^2}{2\sigma^2_{Z|r,\theta}}\right), & \text{for } z \le Z_{\max} \\ 0, & \text{for } z > Z_{\max} \end{cases} \quad (31)$$

where

$$\varepsilon \equiv \frac{2}{1+\operatorname{erf}\left(\frac{Z_{\max}-m_{Z|r,\theta}}{\sqrt{2}\sigma_{Z|r,\theta}}\right)}. \quad (32)$$

Note that $f_{\widehat{Z}}(\widehat{z})$ and $f_Z(z)$ should be regarded as the conditional pdfs given the event of $((r,\theta),\mathrm{BS}_i)$, because both $\widehat{Z}$ and Z are for the user that has been located at (r,θ) and connected to BS_i. Although $f_{\widehat{Z}}(z|(r,\theta),\mathrm{BS}_i)$ and $f_Z(z|(r,\theta),\mathrm{BS}_i)$ are the exact notations, we drop $((r,\theta),\mathrm{BS}_i)$ just for convenience. Note that ε also is a function of (r,θ) because $m_{Z|r,\theta}$ and $\sigma_{Z|r,\theta}$ depend on (r,θ).

Outage probability at each position is given by

$$\begin{aligned}\Pr(\mathrm{Out}|(r,\theta),\mathrm{BS}_i) &= \Pr\left(\widetilde{k} < k_{\min}|(r,\theta),\mathrm{BS}_i\right) \\ &= \Pr(Z > Z_{\max}|(r,\theta),\mathrm{BS}_i) \\ &= 1 - \frac{1}{\varepsilon}. \end{aligned} \quad (33)$$

Outage probability is defined as the averaged outage probability over the region A_0 and can be computed as

$$P_{Out} = \iint_{A_0} \Pr(\mathrm{Out}|(r,\theta),\mathrm{BS}_i) \cdot f(r,\theta|\mathrm{BS}_i)\,dr\,d\theta \quad (34)$$

where $f(r,\theta|\mathrm{BS}_i)$ is computed by using $\Pr(\mathrm{BS}_i|r,\theta)$ and $f(r,\theta)$ in a way that is similar to obtain $f(r,\theta|\mathrm{BS}_i,\mathrm{Avail})$ in (20).

We now derive the pmf of k from the pdf of $\widetilde{k}$. First, considering the applied outage-handling scheme, we replace Z with $\widehat{Z}$ in (26). Then, since $\widetilde{k}$ is a function of $\widehat{Z}$, the pdf of $\widetilde{k}$ can be obtained from $f_{\widehat{Z}}(z)$ as [18]

$$\begin{aligned} f_{\widetilde{k}}(\widetilde{k}|\Xi) &= \frac{f_{\widehat{Z}}(\widehat{z})}{|g'(\widehat{z})|} \\ &= \frac{10\cdot\Lambda\cdot\varepsilon}{\sqrt{2\pi}\cdot\ln 10\cdot\sigma_{Z|r,\theta}\cdot\widetilde{k}^2\cdot\left(\frac{\Lambda}{\widetilde{k}}-\Gamma\right)} \\ &\quad \cdot e^{-\frac{\left(10\log\left(\frac{\Lambda}{\widetilde{k}}-\Gamma\right)-m_{Z|r,\theta}\right)^2}{2\sigma^2_{Z|r,\theta}}}. \end{aligned} \quad (35)$$

Therefore, the pmf of k can be computed as

$$p_k(k|\Xi) = \begin{cases} \int_k^{\frac{\Lambda}{\Gamma}} f_{\widetilde{k}}(x|\Xi)\,dx, & \text{for } k = k_{\max} \\ \int_k^{2k} f_{\widetilde{k}}(x|\Xi)\,dx, & \text{for } k_{\min} \le k < k_{\max} \end{cases} \quad (36)$$

where the integration can be computed by using error functions as

$$\int_{\eta_1}^{\eta_2} f_{\widetilde{k}}(x|\Xi)\,dx = \frac{\varepsilon}{2}\left[\operatorname{erf}\left(\frac{m_{Z|r,\theta}-10\log\left(\frac{\Lambda}{\eta_2}-\Gamma\right)}{\sqrt{2}\sigma_{Z|r,\theta}}\right) - \operatorname{erf}\left(\frac{m_{Z|r,\theta}-10\log\left(\frac{\Lambda}{\eta_1}-\Gamma\right)}{\sqrt{2}\sigma_{Z|r,\theta}}\right)\right]. \quad (37)$$

Consequently, the distribution of the assigned data rate $p_R(R|\mathrm{BS}_i,\mathrm{Avail})$ can be obtained by using (20), (22), and (37).

B. Power-Usage Efficiency per User

The power-usage efficiency per user $\overline{\beta}$ is needed for computation of the pmf $p_R(R|\mathrm{BS}_i,\mathrm{Avail})$. Recall that the given P_i^U cannot always be exhausted because of the data-rate granularity, the peak data-rate limitation, and the power-control operation. Therefore, its actual portion for transmission β is smaller than or equal to 1. The power-usage efficiency $\overline{\beta}$ is defined as the expected value of β.

At each position, β can be represented by a function of $\widehat{Z}$. First, invert (5) as an equality and assume βs for other users in (8) and (9), i.e., $\beta_{i,n}$ and $\beta_{m,n}$, to be $\overline{\beta}$. Next, substitute P^{Ckt} for P_i^{Ckt} and P_m^{Ckt}, and P^U for P_i^U and P_m^U. Then, solving (5) for $\beta_{i,j}$ and dropping the subscripts, we get

$$\beta = q(\widehat{Z})\cdot\left(\Psi + 10^{\frac{\widehat{Z}}{10}}\right) \quad (38)$$

where

$$\Psi \equiv \frac{(1-\phi)\left(\overline{\beta}(1-\alpha-\zeta)\left(1-\frac{1}{N}\right)+\alpha+\zeta\right)}{\overline{\beta}(1-\alpha-\zeta)+\alpha+\zeta} \quad (39)$$

$$q(\widehat{Z}) \equiv \frac{\frac{\left(\frac{E_b}{N_t}\right)_{\mathrm{REQ}}}{\frac{W}{R_0}}\left[\overline{\beta}(1-\alpha-\zeta)+\alpha+\zeta\right]}{\frac{1-\alpha-\zeta}{N\cdot 2^{\left\lfloor \log_2 g(\widehat{Z})\right\rfloor}}\left[1-(1-\phi)\left(2^{\left\lfloor \log_2 g(\widehat{Z})\right\rfloor}-1\right)\frac{\left(\frac{E_b}{N_t}\right)_{\mathrm{REQ}}}{\frac{W}{R_0}}\right]}. \quad (40)$$

The expected value of β at (r,θ), i.e., $E[\beta|\Xi]$, can be obtained by using $f_{\widehat{Z}}(\widehat{z})$.

$$E[\beta|\Xi] = \int_{-\infty}^{Z_{\max}} \beta \cdot f_{\widehat{z}}(\widehat{z})d\widehat{z} = \int_{-\infty}^{Z_{\max}} q(\widehat{z})\cdot\left(\Psi + 10^{\frac{\widehat{z}}{10}}\right)\cdot f_{\widehat{z}}(\widehat{z})d\widehat{z}. \quad (41)$$

Since the quantized value k is given by $2^{\lfloor \log_2 g(\widehat{Z})\rfloor}$, k and $q(\widehat{Z})$ are constant for a certain range of $\widehat{Z}$. Therefore, the integration in (41) can be computed by subdividing the interval as

$$\begin{aligned} E[\beta|\Xi] = & \int_{-\infty}^{10\log\left(\frac{\Lambda}{k_{\max}}-\Gamma\right)} q(\widehat{z})\cdot\left(\Psi+10^{\frac{\widehat{z}}{10}}\right)\cdot f_{\widehat{Z}}(\widehat{z})d\widehat{z}+\cdots \\ & + \int_{10\log\left(\frac{\Lambda}{2k}-\Gamma\right)}^{10\log\left(\frac{\Lambda}{k}-\Gamma\right)} q(\widehat{z})\cdot\left(\Psi+10^{\frac{\widehat{z}}{10}}\right)\cdot f_{\widehat{Z}}(\widehat{z})d\widehat{z}+\cdots \\ & + \int_{10\log\left(\frac{\Lambda}{2k_{\min}}-\Gamma\right)}^{10\log\left(\frac{\Lambda}{k_{\min}}-\Gamma\right)} q(\widehat{z})\cdot\left(\Psi+10^{\frac{\widehat{z}}{10}}\right)\cdot f_{\widehat{Z}}(\widehat{z})d\widehat{z}. \end{aligned} \quad (42)$$

Each integration term can be derived as

$$\begin{aligned} \int_{\gamma_1}^{\gamma_2} q(\widehat{z})\cdot\left(\Psi+10^{\frac{\widehat{z}}{10}}\right)\cdot f_{\widehat{Z}}(\widehat{z})d\widehat{z} = & \frac{\varepsilon\cdot q(\gamma_2)}{2}\left[\Psi(B(\gamma_2,0)-B(\gamma_1,0))\right. \\ & \left. + e^{m_{Z|r,\theta}\frac{\ln 10}{10}+\sigma^2_{Z|r,\theta}\frac{(\ln 10)^2}{200}}(B(\gamma_2,1)-B(\gamma_1,1))\right] \end{aligned} \quad (43)$$

where

$$B(x,y) \equiv \mathrm{erf}\left(\frac{x - m_{Z|r,\theta} - y\cdot\sigma^2_{Z|r,\theta}\frac{(\ln 10)}{10}}{\sqrt{2}\sigma_{Z|r,\theta}}\right). \quad (44)$$

By using the equations above, we can now compute $\overline{\beta}$ as

$$\overline{\beta} = \iint_{A_0} E[\beta|\Xi]\cdot f(r,\theta|\mathrm{BS}_i, \mathrm{Avail})drd\theta. \quad (45)$$

Since the integration in the right side involves $\overline{\beta}$ as well, it is not easy to find the exact solution in closed form. Fortunately, it can be solved by using a numerical method. Because the right-hand side can be represented as a function of $\overline{\beta}$, the previous equation can be rewritten as

$$S(\overline{\beta}) \equiv \overline{\beta} - \iint_{A_0} E[\beta|\Xi]\cdot f(r,\theta|\mathrm{BS}_i, \mathrm{Avail})drd\theta = 0. \quad (46)$$

We can find a solution $\overline{\beta}$ by applying an iteration method such as the *method of false position* to the equation $S(\overline{\beta}) = 0$ (see [19] for a description).

C. Fairness and Throughput

Performance measures for fairness and throughput are defined and derived by using the assigned data-rate distribution that has been obtained in the previous sections. To simplify notation, we will write $p_R(R|\mathrm{BS}_i, \mathrm{Avail})$ as $p_R(R)$ in the following discussion.

We develop a simple but useful formula to model the various time-resource-assignment algorithms, i.e., scheduling algorithms.[5] After the user data rate is determined, transmission duration T is assigned under the rule

$$T = \frac{\tau}{R^\rho} = \frac{\tau}{(k\cdot R_0)^\rho} \quad (47)$$

where ρ is the fairness-control parameter ($\rho \leq 1$) and τ is a constant.

The fairness performance can be controlled with ρ and various scheduling algorithms can be implemented accordingly. For instance, with $\rho = 0$, an equal amount of service time is assigned to all users, i.e., even scheduling. A fair scheduling can also be implemented with $\rho = 1$ because the assigned amount of service time is inversely proportional to the assigned data rate. On the other hand, if ρ is set to an extremely low negative value, almost the entire service time is assigned to one user with the highest data rate, i.e., C/I based scheduling.

We develop and use three different fairness-performance measures for reflecting the QoS discrimination at full loading. The user throughput U can be chosen as the user QoS and is defined as the amount of transmitted data per user during one round-robin period. Therefore

$$U \equiv (1-\mathrm{BLER})\cdot T\cdot R = (1-\mathrm{BLER})\cdot\tau\cdot(k\cdot R_0)^{1-\rho}. \quad (48)$$

The first *fairness-performance factor* $F_{\min/\max}$ is defined as the ratio of the maximum and the minimum of U and can be represented as

$$F_{\min/\max} \equiv 10\log\left(\frac{h_{\min}(U,\chi)}{h_{\max}(U,\chi)}\right) = 10(1-\rho)\log\left(\frac{h_{\min}(R,\chi)}{h_{\max}(R,\chi)}\right) \quad (49)$$

where $h_{\max}(x,\chi)$ denotes the maximum number $\widetilde{x}$ such that $\Pr(x \geq \widetilde{x}) > \chi$ and $h_{\min}(x,\chi)$ does the minimum number $\widetilde{x}$ such that $\Pr(x \leq \widetilde{x}) > \chi$. Therefore, $h_{\max}(x,\chi)$ and $h_{\min}(x,\chi)$ do not imply the absolute minimum and maximum number, but can be regarded as the $(1-\chi)$ percentile and the χ percentile of a discrete number x, respectively. $\Pr(R \geq \widetilde{R}) > \chi$ and $\Pr(R \leq \widetilde{R}) > \chi$ can be represented by

$$\Pr(R \geq \widetilde{R}) = \sum_{R=\widetilde{R}}^{R_{\max}} p_R(R) = \sum_{l=\log_2\left(\frac{\widetilde{R}}{R_0}\right)}^{\log_2(n_{\max})} p_R(2^l\cdot R_0) > \chi \quad (50)$$

[5] In this paper, we have applied a simple scheduling algorithm that determines how much of the time resource should be assigned to a certain user. The order of assignment has not been considered.

$$\Pr(R \le \widetilde{R}) = \sum_{R=R_0}^{\widetilde{R}} p_R(R) = \sum_{l=0}^{\log_2\left(\frac{\widetilde{R}}{R_0}\right)} p_R(2^l \cdot R_0) > \chi \quad (51)$$

where $n_{\max} \equiv R_{\max}/R_0$.

The second fairness-performance factor $F_{\mathrm{mean/STD}}$ based on the mean μ_U and standard deviation σ_U of U is defined and computed as

$$\begin{aligned} F_{\mathrm{mean/STD}} &\equiv 10\log\left(\frac{\mu_U}{\sigma_U}\right) \\ &= -5\log\left(\frac{\sum_{l=0}^{\log_2(n_{\max})}(2^l\cdot R_0)^{2(1-\rho)}p_R(2^l\cdot R_0)}{\left(\sum_{l=0}^{\log_2(n_{\max})}(2^l\cdot R_0)^{1-\rho}p_R(2^l\cdot R_0)\right)^2}-1\right). \end{aligned} \quad (52)$$

The last fairness-performance factor $F_{\mathrm{min/STD}}$ based on the standard deviation and minimum is defined and computed as

$$\begin{aligned} F_{\min/\mathrm{STD}} &\equiv 10\log\left(\frac{h_{\min}(U,\chi)}{\sigma_U}\right) \\ &= 10(1-\rho)\log h_{\min}(R,\chi) \\ &\quad -5\log\left(\sum_{l=0}^{\log_2(n_{\max})}(2^l\cdot R_0)^{2(1-\rho)}p_R(2^l\cdot R_0)\right. \\ &\quad \left. -\left(\sum_{l=0}^{\log_2(n_{\max})}(2^l\cdot R_0)^{1-\rho}p_R(2^l\cdot R_0)\right)^2\right) \end{aligned} \quad (53)$$

We now define *asymptotic throughput* as the expected value of the effective total data rate that a BS transmits at full loading. Let $G(R)$ be the probability that the data rate R occupies radio link of a user; it can then be represented by

$$\begin{aligned} G(R) &= \frac{T\cdot p_R(R)}{\sum_{R=R_0}^{R_{\max}} T\cdot p_R(R)} \\ &= \begin{cases} \dfrac{R^{-\rho}p_R(R)}{\sum_{l=0}^{\log_2(n_{\max})}(2^l\cdot R_0)^{-\rho}p_R(2^l\cdot R_0)}, & \text{for } R=2^s\cdot R_0 \\ 0, & \text{elsewhere} \end{cases} \end{aligned} \quad (54)$$

where $s = 0, 1, 2, \ldots, \log_2(n_{\max})$. Total transmitting data rate of a BS, i.e., C, is the sum of N data rates occupying the radio links. These N data rates are assumed to be independent from one another. The effective data rate can be represented by using BLER, as in [20]. Therefore

$$\begin{aligned} &\text{Asymptotic throughput} \\ &\equiv \frac{E[C]}{1+\sum_{i=1}^{\infty}\mathrm{BLER}^i} \\ &= (1-\mathrm{BLER})\cdot N\cdot \sum_{l=0}^{\log_2(n_{\max})} 2^l\cdot R_0\cdot G(2^l\cdot R_0). \end{aligned} \quad (55)$$

TABLE I
SYSTEM PARAMETERS AND DEFAULT VALUES

Parameters		Values
Chip rate (Mcps)		3.84
Modulation		QPSK
Channel code rate		1/3
Rate Set I	SF set	{256, 128, 64, 32, 16, 8, 4}
	Data rate set (kbps)	{10, 20, 40, 80, 160, 320, 640}
Rate Set II	SF set	{128, 64, 32, 16, 8}
	Data rate set (kbps)	{20, 40, 80, 160, 320}
$(E_b/N_t)_{REQ}$ at R_0 (10 kbps)		3.0 (dB) for BLER = 0.1
Path loss exponent, μ		4
Shadowing standard deviation, σ		10
Site-to-site correlation, b^2		0.5
Downlink orthogonality, ϕ		0.5
Overhead channel power ratio, α		0.2
Circuit channel power ratio, σ		0.4
Criterion for the max and the min, χ		0.01

IV. NUMERICAL RESULTS

The analysis developed in the previous section is applied to a system model based on 3GPP wide-band code-division multiple-access (WCDMA) frequency-division duplex (FDD) release 1999 (R99) system. Numerical results of the performance measures are presented.

A. 3GPP WCDMA System Model

Downlink shared channel (DSCH) in a 3GPP WCDMA FDD R99 system is selected as the packet channel model [21].[6] The system parameters and default values set for the analysis are listed in Table I.

In order to present the effect of the limited data-rate set on the system performance, we apply two different rate sets and compare their performances. SFs and data rates for each rate set are listed in Table I. To ensure fairness in the comparison, we set the data rate for the largest SF R_0 to be the same as 10 kb/s for both rate sets. The same $(E_b/N_t)_{\mathrm{REQ}}$ is then assumed at R_0, so that the identical link performance can impartially be applied to the two rate sets. Instead, the minimum number of equivalent code channels $k_{\min}$ is set to 1 for Rate Set I, but $k_{\min}$ to 2 for Rate Set II, considering the actual largest SF.

The cellular system is modeled by locating BSs at the centers of hexagonal grid pattern, as shown in Fig. 2. An omni directional antenna pattern is used. To simplify the analytical procedure, the A_0 region is restricted to the sector of radius D_A and of angle $\pi/6$. Note that by symmetry, the relative position of users and BSs is the same throughout as for the sector of the

[6]In this paper, we have assumed that the system performance may not be affected by the limitation of available orthogonal VSF (OVSF) codes. In the extreme cases of the indoor or pedestrian environments with a high orthogonal link, the limited number of OVSF codes may impose a limit on the number of simultaneous users.

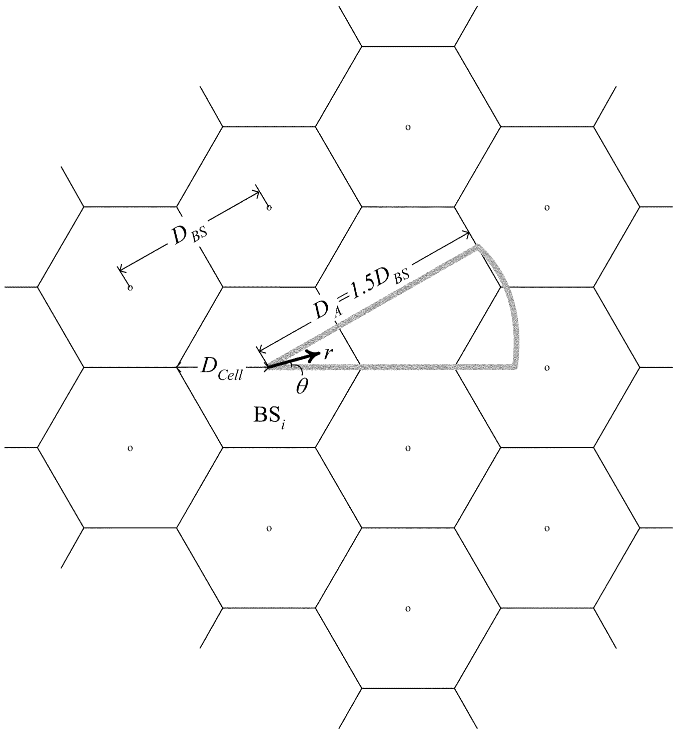


Fig. 2. Cell sites deployment model.

figure. We apply the uniform distribution to the user location. Therefore

$$f(r,\theta)=\begin{cases}\frac{12r}{\pi D_A^2}, & \text{for } 0<r\le D_A \text{ and } 0\le\theta<\frac{\pi}{6}\\ 0, & \text{elsewhere}\end{cases}. \quad (56)$$

Let us summarize the analytical procedure. With the values given in Table I and the user distribution above, we first compute the power-usage efficiency $\overline{\beta}$ by using the numerical integrations and iteration method, as shown in Section III-B. Next, we derive the pmf of the assigned data rate $p_R(R)$ by using the computed $\overline{\beta}$, as shown in Section III-A. *Outage probability* P_{Out} is also computed at this stage. Finally, we obtain the three *fairness-performance factors* and the *asymptotic throughput* by using the derived $p_R(R)$, as shown in Section III-C.

We have also developed a computer simulation to validate the analytical results. The *Monte Carlo* simulation model consists of 19 cells of two tiers. We collect data from the center cell for statistics.

B. Analytical Results and Simulation Results

Before proceeding to the system-performance evaluation, we show the effect of the best BS selection on the distribution of the connected users. The user distributions with and without the best BS selection are derived and compared. For simplicity, we consider the region of $0\le\theta<\Delta\theta$ and assume that the outage probability is 0. Provided that $\Delta\theta$ is sufficiently small, the distribution with the best BS selection can be computed by using $f(r,\theta|\mathrm{BS}_i,\text{Avail})$ as

$$f(r|0\le\theta<\Delta\theta,\mathrm{BS}_i,\text{Avail})\approx\frac{f(r,\theta=0|\mathrm{BS}_i,\text{Avail})}{\int_0^{D_A}f(r,\theta=0|\mathrm{BS}_i,\text{Avail})dr}. \quad (57)$$

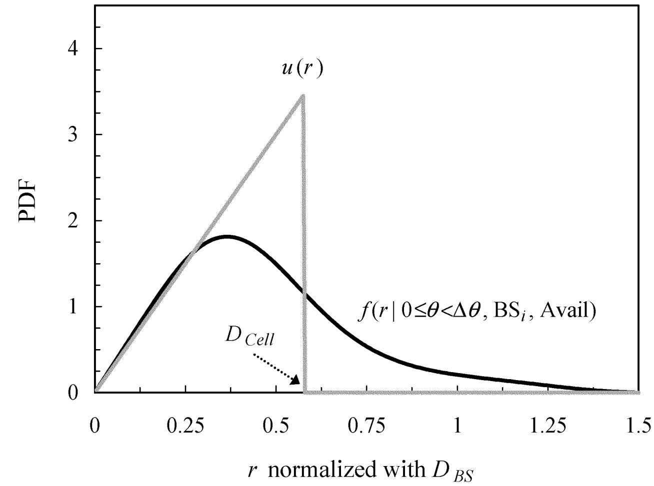


Fig. 3. Distribution of the connected user to a BS.

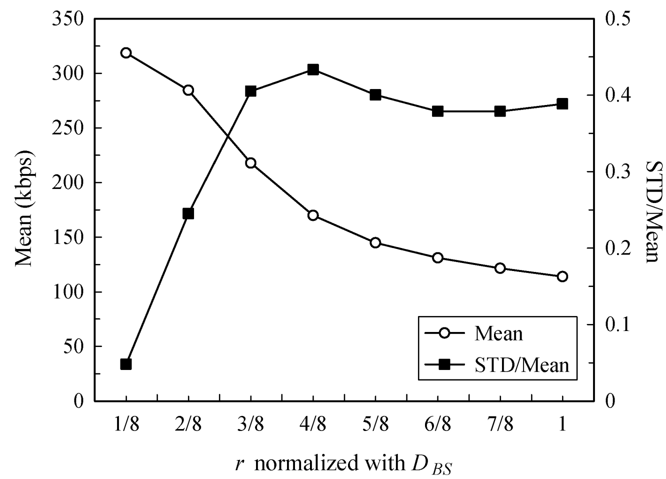


Fig. 4. Assigned data rate at various locations: Rate Set I, $N=4$.

Without the best BS selection, the distribution of users selecting the closest BS can be given by

$$u(r)\approx\begin{cases}\frac{2r}{D_{\text{Cell}}^2}, & \text{for } 0<r\le D_{\text{Cell}}\\ 0, & \text{elsewhere}\end{cases}. \quad (58)$$

The two distributions above are plotted in Fig. 3. This figure shows the difference between the two distributions. In particular, we can see that the distribution with the best BS selection in the region of $r>D_{\text{Cell}}$ is too large to be neglected. Therefore, this result confirms that the effect of the best BS selection on the user distribution should be considered for the exact analyses.

The statistics of the assigned data rate R at various locations are shown in Fig. 4. Its mean and standard deviation are obtained from $p_k(k|\Xi)$ in Section III. We set θ to 0 and r to a multiple of $D_{\text{BS}}/8$. As MS is farther away from BS, the mean is decreased due to higher path attenuation. From a distance of $D_{\text{BS}}/2$, however, the normalized standard deviation is not increased and the curve of the mean becomes less steep. This is because the best BS selection reduces more variance of Z at that region, i.e., the macro diversity effect. Fig. 4 shows that the assigned data rate is dependent on user location. This feature is the main cause of the downlink tradeoff to be shown in the following.

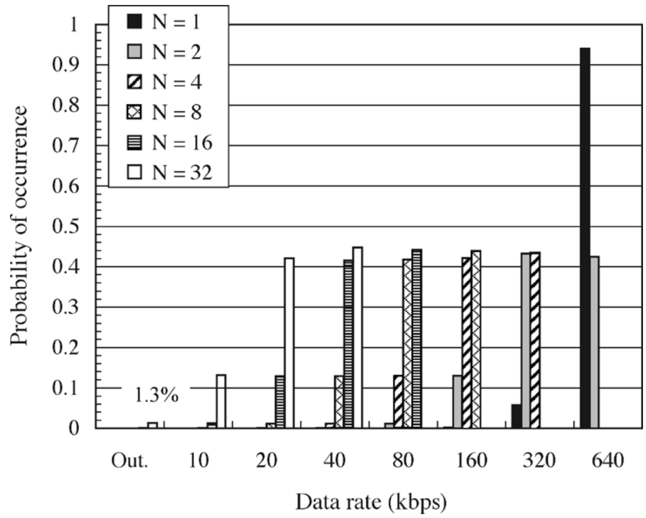


Fig. 5. The pmf of the assigned data rate: Rate Set I.

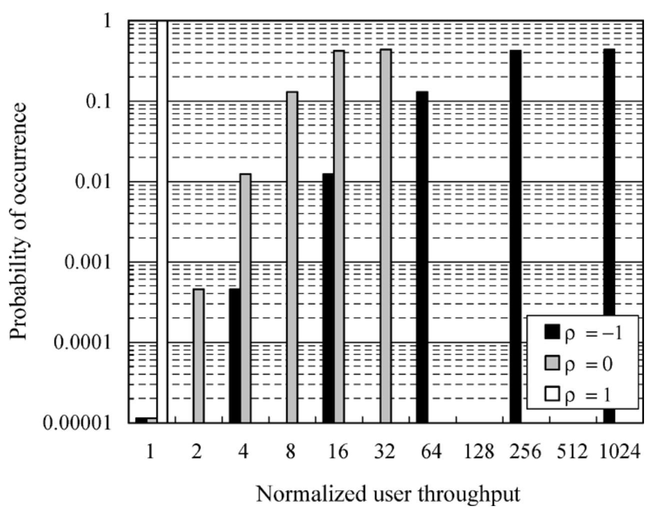


Fig. 6. The pmf of user throughput: Rate Set I, $N = 4$.

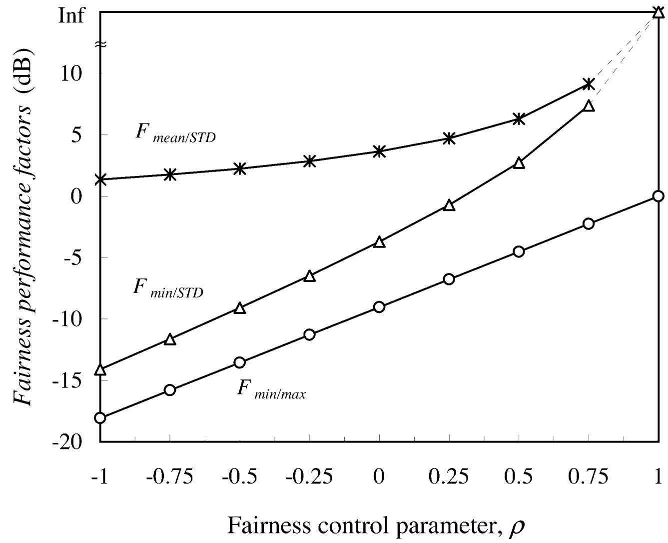


Fig. 7. Fairness-performance factors: Rate Set I, $N = 4$.

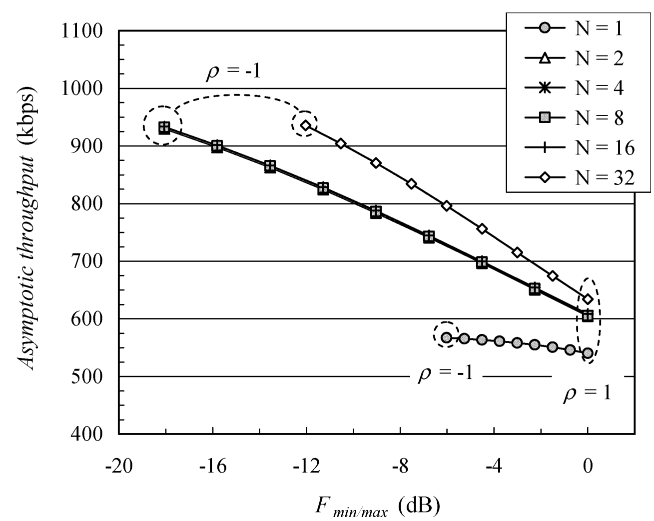


Fig. 8. Tradeoff between throughput and fairness: Rate Set I.

Fig. 5 shows the pmf of the assigned data rate to a user $p_R(R)$ for various the numbers of simultaneous users N. As N is increased, the maximum allowable power per user P^U is decreased and then the data-rate distribution moves toward a lower data rate. When $N = 1$, the highest assigned data rate is limited by the predetermined rate set, rather than the interference power. Therefore, $p_R(R = 640\ \mathrm{k})$ for $N = 1$ is abruptly higher than the others. On the other hand, when N is more than or equal to 32, outage events occur due to too small P^U.

We normalize the user throughput U with $(1 - \mathrm{BLER}) \cdot \tau \cdot (k_{\min} \cdot R_0)^{1-\rho}$ and show its pmf in Fig. 6. When the fairness control parameter ρ is set to 1, U is constant regardless of the assigned data rate R. This implies that a BS provides the same throughput to every data user with fair scheduling. As ρ is decreased, the distribution becomes wider and fairness performance would then be degraded. In Fig. 7, we will show that the degree of fairness can be controlled by ρ.

Fig. 7 shows the three *fairness-performance factors* with various settings of ρ. We can observe that each of the factors increase monotonically as ρ is set to be higher. Therefore, it is found that the three fairness-performance factors reflect the fairness performance well. However, with $F_{\mathrm{mean/STD}}$ or $F_{\min/\mathrm{STD}}$, it is inconvenient to plot the perfect fair point being reached by setting $\rho = 1$. For this reason, we select and use $F_{\min/\max}$ as the fairness measure in the following analyses.

The quantitative tradeoff relation between throughput and fairness is shown in Fig. 8. *Asymptotic throughput* is plotted as a function of $F_{\min/\max}$ with the set of $\{\rho : -1, -0.75, \ldots, 0.75, 1\}$. As expected, asymptotic throughput is increased with lower ρ. Hence, this figure confirms that asymptotic throughput and $F_{\min/\max}$ have a tradeoff relation and the tradeoff can be controlled by ρ.

Fig. 8 also shows that throughput can be increased with N. When $N = 1$, throughput is the lowest because of the peak data-rate limitation. On the other hand, throughput for $N = 32$ is the highest because of the resource reallocation of outage users. This improvement is, however, achieved at the expense of service availability. Outage probability for $N = 32$ is about 1.3%, as shown in Fig. 5. For $2 \leq N \leq 16$, the curves are almost the same because the assigned data rate is not affected by the peak data-rate limitation and also because the outage event does not occur. As N is increased, the multiuser interference is

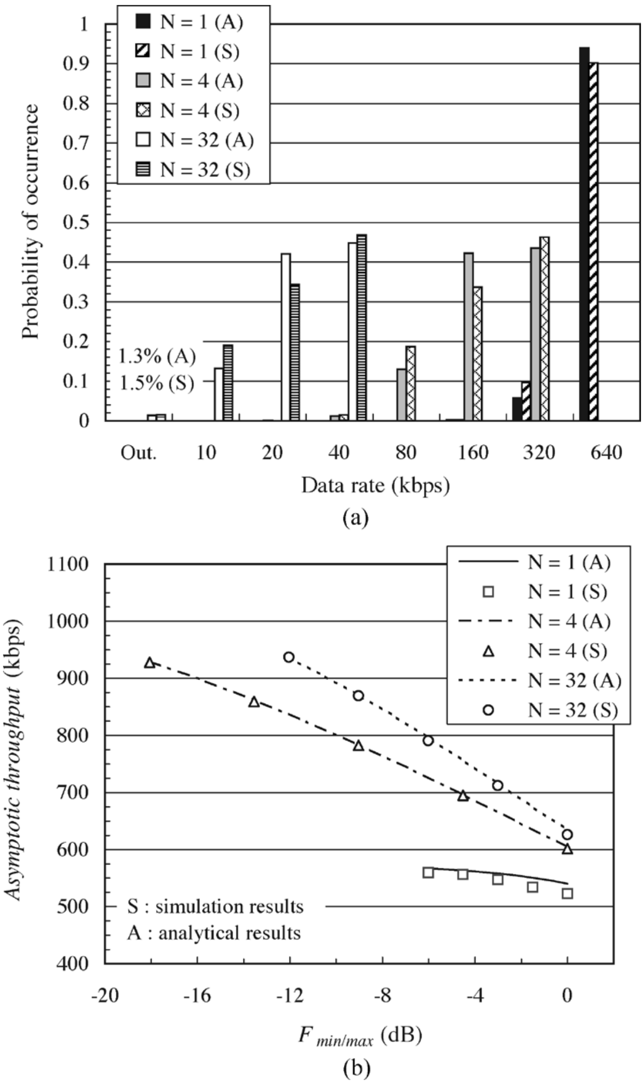


Fig. 9. Comparison of analytical results and simulation results: Rate Set I. (a) The pmf of the assigned data rate. (b) Tradeoff between throughput and fairness.

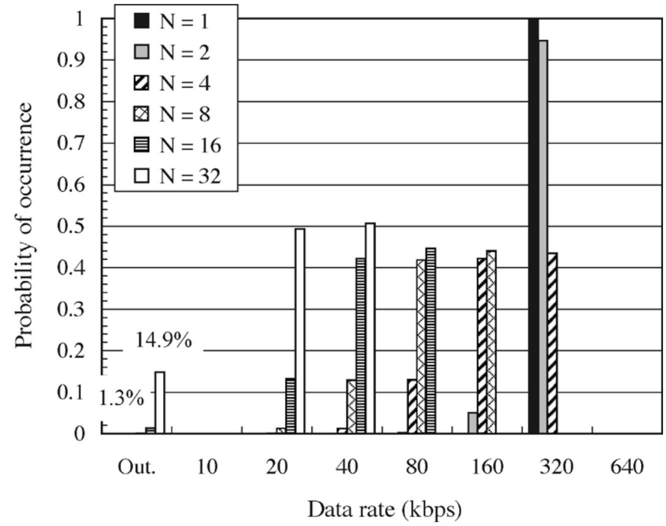


Fig. 10. The pmf of the assigned data rate: Rate Set II.

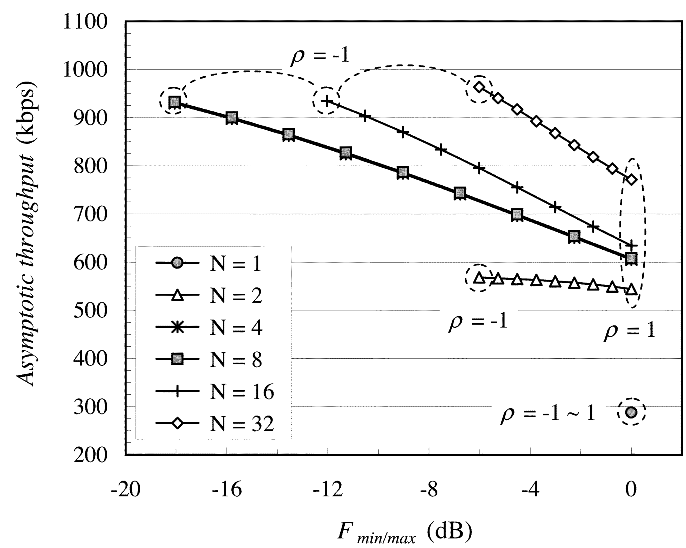


Fig. 11. Tradeoff between throughput and fairness: Rate Set II.

increased, but the self interference is decreased. As mentioned in Section II-A, Latva-aho [12] showed that the link-level degradation caused by the self-interference is approximately equal to that by the multiuser interference. Since the received E_b/N_t model in this paper has been developed based on the results of such a link-level study, our results show that the throughput for $2 \leq N \leq 16$ is the same regardless of N. The effect of the self-interference on the system throughput has been analyzed in detail in our previous work [22].

The simulation and analytical results are shown and compared in Fig. 9. To make the analytical model simple, the random variable β is assumed to be a constant value $\overline{\beta}$ in Section III. The value of β for each user varies randomly according to the user location as well as shadowing experienced. This randomness of β is applied in our simulation program. Though the analytical model has been employed this simplification, we can see in Fig. 9(a) that the analytical $p_R(R)$ and *outage probability* agree with the simulation results.

Both $F_{\min/\max}$ and *asymptotic throughput* in the analytical procedure are obtained from $p_R(R)$, but these performance measures in the simulation are statistically computed based on the collected user throughput U and total data rate C, respectively. Fig. 9(b) shows that the simulation and analytical results for both performance measures match well. The comparison in Fig. 9, therefore, confirms that we can use the proposed analytical model and method in this paper with confidence.

The pmf of the assigned data rate for Rate Set II is shown in Fig. 10. Compared with the results for Rate Set I in Fig. 5, the peak data-rate limitation more affects the data-rate distribution. In Fig. 10, we can see that the highest assigned data rate is subject to the upper limitation of rate set, even for $N = 2$. *Outage probability* is also increased due to the rise of the lower limitation.

The tradeoff between *asymptotic throughput* and $F_{\min/\max}$ for Rate Set II is shown in Fig. 11. The tradeoff performance is improved with N, similar to the results for Rate Set I. However, the narrower rate set causes more diverse tradeoff curves. When $N = 1$, asymptotic throughput for Rate Set II is reduced by about 49% as compared to that for Rate Set I in Fig. 8. On the other hand, *outage probability* of 14.9% for $N = 32$ gives more throughput. It may not be desirable to set $N = 32$ because such high outage probability cannot be allowed. If 1.3% outage probability can be tolerated, we can maximize throughput and fairness performance by setting N to 16. Figs. 8 and 11 show that the tradeoff performance can be influenced by the given rate set and the setting of N. Therefore, control parameters N and ρ should be cautiously and properly selected, considering the given rate set and the criterion for throughput and fairness.

V. Conclusion

An analytical model and a method have been developed for the tradeoff analysis between throughput and fairness on the multirate CDMA packet downlinks. The proposed model and the method reflect the effects of the multiplexing scheme, the scheduling scheme, the limited data-rate set, log-normal shadowing, the best BS selection, and the self-interference. The numerical results have also been derived based on the 3GPP WCDMA R99 system model.

The quantitative tradeoff relation between asymptotic throughput and fairness-performance factors has been derived analytically. The assigned data rate depends on the user location, as expected. Then, discrimination against users with low C/I channel conditions leads to an increase in system throughput, but the fairness can be degraded at the same time. It is shown that the degree of fairness can be controlled by the fairness-control parameter of the proposed scheduling model.

We have also obtained the tradeoff relations for various numbers of simultaneous users. The results show that fairness-performance factors and asymptotic throughput can be improved with a greater number of simultaneous users. It is found that CDS may outperform TDS with respect to the tradeoff when taking account the self-interference, peak data-rate limitation, and resource reallocation of users in the outage.

The effect of the limited data-rate set on the tradeoff has been derived and presented. As the narrower rate set is applied, the tradeoff becomes more sensitive to the setting of the number of simultaneous users and then more diverse tradeoff curves are produced. This implies that the tradeoff can be improved if the number of simultaneous users is set properly with consideration of the given rate set.

To confirm the analysis results derived, we developed a computer simulation. The comparison shows that the analytical and simulation results are in very good agreement and confirms that we can use the analytical method proposed in this paper with confidence.

In this paper, we have analytically evaluated the performances of the CDMA packet downlink with location-dependent C/I. Although we have analyzed the system performance at the full loading, the results shown in this paper would be helpful in developing a scheduling scheme that can efficiently handle the bursty traffic. It is also believed that the presented model and method can be utilized for predicting the performance of CDMA downlink packet systems.

References

[1] M. Airy and K. Rohani, "QoS and fairness for CDMA packer data," in *Proc. 51st IEEE VTC'00*, Tokyo, Japan, May 2000, pp. 450–454.

[2] P. Bender *et al.*, "CDMA/HDR: A bandwidth-efficient high-speed wireless data service for nomadic users," *IEEE Commun. Mag.*, vol. 38, pp. 70–77, July 2000.

[3] H. Holma and A. Toskala, *WCDMA for UMTS*. New York: Wiley, 2000.

[4] I. Lopez *et al.*, "Downlink radio resource management for IP packet services in UMTS," in *Proc. 53rd IEEE VTC'01*, Rhodes, Greece, May 2001.

[5] J. M. Holtzman, "Asymptotic analysis of proportional fair algorithm," in *Proc. 12th PIMRC*, San Diego, CA, Oct. 2001, pp. F-33–F-37.

[6] V. Bharghavan *et al.*, "Fair queuing wireless networks: issues and approaches," *IEEE Pers. Commun. Mag.*, vol. 1, pp. 44–53, Feb. 1999.

[7] A. Bedekar *et al.*, "Downlink scheduling in CDMA data networks," in *Proc. IEEE GLOBECOM'99*, Rio de Janeiro, Brazil, Dec. 1999, pp. 2653–2657.

[8] C. Mihailescu *et al.*, "Radio resource management for packet transmission in UMTS WCDMA system," in *Proc. 50th IEEE VTC'99*, Amsterdam, The Netherlands, Sept. 1999, pp. 573–577.

[9] 3GPP TR 25.922, Radio resource management strategies, Release 1999, Ver. 3.5.0, 3GPP, p. 37, Mar. 2001.

[10] 3GPP TR 25.214, Physical layer procedure (FDD), Release 1999, Ver. 3.7.0, 3GPP, June 2001.

[11] E. Dahlman and K. Jamal, "Wide-band services in a DS-CDMA based FPLMTS system," in *Proc. 46th IEEE VTC'96*, Atlanta, GA, Apr. 1996, pp. 1656–1660.

[12] M. Latva-aho, "Bit error probability analysis for FRAMES WCDMA downlink receivers," *IEEE Trans. Veh. Technol.*, vol. 47, pp. 1119–1133, Nov. 1998.

[13] A. J. Viterbi *et al.*, "Soft handoff extends CDMA cell coverage and interferences reverse link capacity," *IEEE J. Select. Areas Commun.*, vol. 12, pp. 1281–1288, Oct. 1994.

[14] M. Pratesi *et al.*, "Outage analysis mobile radio systems with generally correlated log-normal interferers," *IEEE Trans. Commun.*, vol. 48, pp. 381–385, Mar. 2000.

[15] P. R. Patel *et al.*, "A simple analysis of CDMA soft handoff gain and its effect on the cell's coverage area," in *Proc. 15th WINLAB Workshop*, East Brunswick, NJ, Apr. 1995, pp. 101–118.

[16] M. Zorzi, "On the analytical computation of the interference statistics with applications to the performance evaluation of mobile radio systems," *IEEE Trans. Commun.*, vol. 45, pp. 103–109, Jan. 1997.

[17] J. Cho and D. Hong, "Statistical model of downlink interference for the performance evaluation of CDMA systems," *IEEE Commun. Lett.*, vol. 6, pp. 494–496, Nov. 2002.

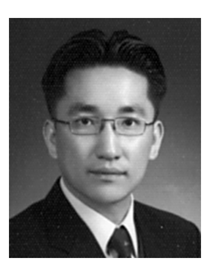
[18] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*, 3rd ed. New York: McGraw-Hill, 1991.

[19] M. Ahmed *et al.*, "Spectral efficiency estimates for CDMA2000—a third generation cellular standard," in *Proc. 49th IEEE VTC'99*, Houston, TX, May 1999, pp. 361–365.

[20] 3GPP TR 25.211, Physical channels and mapping of transport channels onto physical channels (FDD), Release 1999, Ver. 3.7.0, 3GPP, June 2001.

[21] J. Cho and D. Hong, "Throughput comparison of CDM and TDM for downlink packet transmission in CDMA systems with a limited data rate set," *IEEE Trans. Wireless Commun.*, vol. 3, pp. 600–611, Mar. 2004.

[22] E. Kreyszig, *Advanced Engineering Mathematics*, 8th ed. New York: Wiley, 1999.



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